

Elastic properties of vanadium pentoxide aggregates and topological defects

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We study the aqueous solution of vanadium pentoxide by using topology methods. The experiments by Zocher, Kaznacheev, and Dogic exhibited, that in the sol phases of $V_2O_5 - H_2O$, the tactoid droplets of V_2O_5 can coalesce. In the magnetic field, this effect is associated with a gauge field action, viz. we consider coalescence (in the topologically more convenient term, "junction") of droplets as annihilation of topological defects, concerning with the tactoid geometry. We have shown, that in the magnetic field, the tactoid junction is mainly caused by non-Abelian monopoles (vortons), whereas the Abelian defects almost do not annihilate. Taking into account this annihilation mechanism, the estimations of time-aging of the $V_2O_5 - H_2O$ sols may be specified.

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I. INTRODUCTION

The tactoid sol phase of the $V_2O_5 - H_2O$ system has been discovered at the 20-th years of the last century by Zocher (see references in [1, 2]). At the beginning of our century, the tactoid drops (tactoids) have been investigated on the optical experiments by Kaznacheev [2], Lavrentovich [3], Dogic (see [4] and references in [5]), and their coworkers. The tactoid phase is chemically classified as the lyotropic inorganic nematic [1]. The tactoids coexist with the isotropic liquid phase at the mass concentration of V_2O_5 , amounting 0.3-2.1 percents, and under other standard conditions [2].

The thermodynamic parameters and pH cause the dynamics of their formation, in particular, the junction.

The tactoid geometry is evolved complicatedly (and mutually inversely) in depending on time-aging of the sols [2].

Due to the de Gennes's theory [6], the tactoid shape stabilization is defined by competition between the elastic energy of the nematic phase, the surface energy, and the anchoring energy [2]. The minimum of the tactoid free energy provides an equilibrium shape of a droplet. The measured macroscopic elastic moduli are in a very large ratio ($\frac{K_3}{K_1} > 100$), that distinguishes $V_2O_5 - H_2O$ from other lyotropic liquid crystals (LC), whose typical values of $\frac{K_3}{K_1}$ are in order of ten.

In the magnetic field, the prolate droplets are aligned by their long axes parallel to the field. Then the special case of the junction of tactoid poles may be observed [1, 2].

Remarkably, that the sol phases of $V_2O_5 - H_2O$ were conditionally sorted on a shape polarity and a nematic director field [5] as of a homogeneous and a non-uniform field, and of the spherical and the bispherical [2] drops with boojums. Strikingly simultaneously, these phases have been parsed (see [5, 7, 8] and references therein) basing on the experiments by Dogic (references in [5]), performed independently of Kaznacheev.

In this paper, we study the mesomorphism of the $V_2O_5 - H_2O$ system during the tactoid junction and spec-

ify the character of the mesomorphic consequence there. Our goal is to define the influence of junction onto dynamic parameters of the sol system, including time-aging of the sols. In addition, aging of these sols in water is an applied problem of ecology, since V_2O_5 contains in coal impurities, generated in result of work of thermal power stations.

From a topological standpoint, poles of a tactoid are the point defects, boojums. As will readily be observed, we have to do with a quantum phase transition, the analogous topological singularities of two poles (each admitting a flux) were announced by Haldane [9] for the quantum Hall semiconductors. Also, there is a convenient analogy with the boojum formalism for the superfluid phases of 3He and 4He [10], however their varied topology descriptions does not allow to explain the case of the tactoid coalescence.

II. FORMALISM

Geometry of the droplets obeys the local nematic order parameter \mathbf{n} , which is oriented relatively to a droplet surface (Fig. 1).

The free energy functional of a tactoid in the magnetic field is summed up from the Frank elastic energy F_{el} and the magnetic energy F_m [2, 5]:

$$F = F_{el} + F_m, \quad (1)$$

$$F_{el} = \int_V d^3\mathbf{r} \left[\frac{K_1}{2} (\nabla \cdot \mathbf{n})^2 + \frac{K_2}{2} (\mathbf{n} \cdot \nabla \times \mathbf{n})^2 + \frac{K_3}{2} [\mathbf{n} \times (\nabla \times \mathbf{n})]^2 - K_{24} \nabla \cdot [\mathbf{n} \cdot \nabla \mathbf{n}] \right] \quad (2)$$

The magnetic energy density has the form $-\frac{\chi_a}{2} (\mathbf{n} \cdot \mathbf{H})^2$, (where χ_a is the anisotropy of magnetic susceptibility, and \mathbf{H} is the magnetic field).

The terms at K_1 , K_2 , and K_3 elastic constants in 2 mean splay, twist, and bend deformations of a bulk nematic respectively, \mathbf{n} is the coordinate dependent nematic director. The term at K_{24} relates to saddle-splay deformation mode [5]. In this continuum, the tactoid boojums

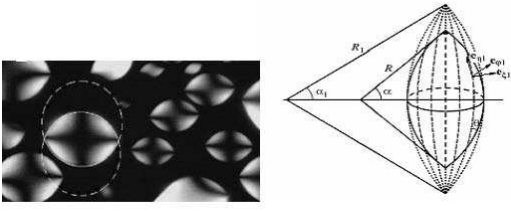


FIG. 1: The director field on the tactoid surface, taken over [12]. R_i and α are the geometric parameters, $\gamma = (\frac{\tan(\alpha/2)}{\tan(\alpha/2)})^2$, $0 \leq \gamma \leq 1$, the vectors \mathbf{e}_i $i = \varphi_{kazn}, \xi, \eta_{kazn}$ denote the bisppherical coordinates.

$$4\pi(\sin \alpha - \alpha \cos \alpha) + \pi(3 \sin \alpha - 3\alpha \sin \alpha - \alpha^2 \sin \alpha) + \pi \sin^3 \alpha \int_{-\infty}^{\infty} \frac{\sin \theta}{(\cosh \eta_{kazn} + \cos \alpha)} d\eta_{kazn} + \frac{\pi}{36}[\sin \alpha(20 + \cos \alpha) - 3\alpha \cos \alpha(7 + 2 \sin^2 \alpha)] \quad (3)$$

here θ is the parameter with the too long dependence of α , γ , η_{kazn} [13], the last term of (4) corresponds to the magnetic energy at $\gamma \rightarrow 1$. For γ , see Fig.1.

Nematic surface defects of the tactoids [12] are of the homotopic group $\pi_2(R, \tilde{R}) = P \times Q$, the defects of the P group are living only at the surface (P group is the kernel of the homomorphism $\pi_1(\tilde{R}) \rightarrow \pi_1(R)$ and consists of integers [3]), and Q 's defects are arrived from the interior. (Here R and \tilde{R} denote the space of degenerate states in the volume and the non-vanishing states on the surface, which are arrived from the interior, respectively). The interior may be inhabited by hedgehogs. All of these point defects keep within the exact homotopic sequence [12]:

$$\pi_2(\tilde{R}) \longrightarrow \pi_2(R) \longrightarrow \pi_1(\tilde{R}) \longrightarrow \pi_1(R). \quad (4)$$

Boojums are characterized by topological charges m and n [3], which depend on a configuration of a nematic director's field. Annihilation of the boojums of the adjacent tactoids does not mean an influence of the raising hedgehog's (in topology, they are not arbitrary floating to the tactoid surface). Kurik and Lavrentovich [14] have mentioned about some strings, connecting opposite boojums via a hedgehog in nematic droplets, however, non-triviality of π_1 group hampered the revealing of the droplet junction without the disclination concept. However, in our case we reasonably ignore lacking disclinations (see the conclusions by Balachandran et al. [15]).

Interaction scales are the 'dipole length' L_{dip} , and the 'correlation length' L_ξ [16], which are characterized an

were revealed by Kaznacheev [2] and by van der Schoot [5] practically identically, independently of one another. The final result of tactoid classification is the existence of four regimes of form is possible, which depend on anchoring between the local director and the tactoid surface, and also on the total tactoid volume [11]. Only at the weak-coupled limit ($\gamma = 0$), Kaznacheev found an equilibrium shape of a tactoid [2], [13], *a fortiori* at $F_m = 0$ and without the terms of K_{24} -s in (2). At the limit (Fig. 1), the free energy (1) is the almost non-analytical function on $f(\alpha, \gamma)$ [2, 13]:

action of the group of the order parameter. We assume L_{dip} is in connection with a long-axis of a tactoid.

In the Cartesian coordinates (x, y, z) , the director field has the configuration $\mathbf{n} = n(0, 0, \frac{1 - \cosh \eta_{kazn} \cos \xi}{\cosh \eta_{kazn} - \cos \xi})$, where η_{kazn}, ξ are the bisppherical coordinates [2].

Quite evidently, that tactoid system is provided by a gauge field [17] (and a field with $SU(2)$ symmetry). Concerning an universality class of the system, take the V_2O_5 droplet surface as belonging to $SO(3)$ group of rotations of the two-dimensional sphere (here 'tactoid') S^2 [16]. $U(1)$ will a group of rotations around a droplet axis, which is agree closely with the magnetic phase group of $^3He - A$ [18]. $U(1)$'s winding is realized of non-trivial topology of tactoids.

The $SO(3)$ and $SU(2)$ groups are locally isomorphic (as their Lie algebras) and are connected by the homomorphism, $SO(3) \sim SU(2)/Z_2$, where our Z_2 is the boojum's boundary condition.

In our standpoint, at the bulk junction, the group $SO(3)^n \times U(1)^{2n}$ broken down to $SO(3)^{n-1} \times U(1)^{2n-1}$, where n is a number of tactoids.

A model of the sol should involve the monopole solutions, according to the theorem [16] about requirement of their existence ($\pi_2(G/H) \longrightarrow \pi_1(H)$).

On the other hand, inasmuch as $\pi_1(H) = \underbrace{Z \otimes Z \otimes Z \otimes \dots \otimes Z}_{2n}$, the $V_2O_5 - H_2O$ sols are of the group G . The tactoid annihilation may be described either by non-Abelian or Abelian theory in depending on the global field $SU(2)$. Besides, we have to expect ap-

pearance of a compensative vector field [19].

Here, an each tactoid, in correspondence to two poles (boojums) on a tactoid surface, may contain two vortons with their tails (the wide and "over-Witten's" definition for vortons see in [20, 21, 22], this is a kind of monopoles with the definite pair of topological charges, vortex and azimuthal windings). Just as vortices, they appear, if the order parameter has extra degrees of freedom besides of the overall phase [23, 24]. In the tactoid free energy, the terms of twisted deformations [8] may play a role in these excitations. By introducing a necessary parametrization, the free energy equation, analogous to [2], was proposed in [8], where the free parameters permit to be the non-commutative relations in the droplet symmetry. Let us note, that we use the factor-space CP^1 in accordance to a chiral (gauge) field (2) [17].

Though, due to the electromagnetic (no topological) reasons, the sol tactoids can survive coalescence owing to the Coulomb attraction in water. But from topology [14], we do not yet know about appearance of a physical field from the configuration of defects. We have to note, that because of in-homogeneity of a system, we have a wide class of string models for a prototype.

III. ANNIHILATION OF TOPOLOGICAL DEFECTS

So, a junction of droplets means, that the surface point defect (boojum) configuration may be unstable ($\gamma \neq 0$). We discuss the Abelian and non-Abelian string configurations [18, 25, 26, 28, 29], which support the sols of tactoid nematics. Their combinations and interactions are expected to define of the junction of tactoids.

A. Abelian space

The Abelian character of pair boojums and monopoles, and also their integer charge were proven [10]. Boojums of charge $N = \pm 1$ live at $L_\xi \ll L \ll L_{dip}$ [16, 20]. But from the surface field phenomenology [2] of a solitary tactoid, one can not define a flux number k [16], concerning an each boojum, only what $k = 1$ is preferable for their pairing configuration, and $k = 2$ for a unit singularity. In this scenario, annihilation of charge-opposite (topological) 'particles' is possible.

Abelian monopoles may be associated with locations of boojums, but, due to the topological properties of our G , we ignore them. Let us consider only vortons of the Abelian gauge. They are unstable [26], and appear together with the neutral strings. The open question is which velocity will greater: of the tactoid coalescence or the vorton decay.

In the $U(1)$ gauge, the loop-radius dependent criterion of the vorton stability was found and analyzed numerically in the case of the potential expressed in the elliptic ansatz [27], as well as in the well-known Witten's

$U(1) \times U(1)$ case (see review [21], that is an analogous phase transition from $U(1) \times U(1)$ to $U(1)$ for two neighbouring randomly oriented tactoids, in absent of magnetic field.

B. Non-Abelian space

Usual Lagrangians of non-Abelian theories are often linearized into the Bogomolny-Prasad-Sommerfeld (BPS) equations [30]. A number of applications corresponding to similar strings were considered, for example, in [15, 21, 22, 24, 25, 28, 29, 31, 32].

In the phase diagram [8], the regions of twist states were indicated. If the tactoid junction carry out there, for spherical and prolate droplets, one may make an analogy between the non-Abelian vortons and "rotation" of the nematic order parameter, in spite of the ansatz ($\alpha(\eta) = \alpha_0 \sin \eta$ [8]) condition, labeled one of the topological invariants.

Let us formulate the string model with the boson Lagrangian density (due to [21, 22])

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu} - D_\mu\vec{\phi}^\dagger \cdot D^\mu\vec{\phi} - V(\vec{\phi}). \quad (5)$$

Here

$$F^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (6)$$

are the Abelian field strengths. The global curvature is

$$G_{\mu\nu} = \partial_\mu \mathbf{H}_\nu - \partial_\nu \mathbf{H}_\mu + g\mathbf{H}_\mu \times \mathbf{H}_\nu. \quad (7)$$

The gauge covariant derivatives of vacuums are:

$$D_\mu(\vec{\phi}) = \partial_\mu\vec{\phi} - ieA_\mu\vec{\phi} + g\mathbf{H}_\mu \times \vec{\phi}, \quad (8)$$

In the formulas (5) - (8), μ and ν are indices of the gauge field A and of the metrics g . H_μ and ϕ are the three-dimensional vectors in the $SU(2)$ Lie algebra. The field potential $V(\phi)$ is expressing from (2). Due to [13]

$$x = a \frac{\sin \xi \cos \varphi}{\cosh \eta - \cos \xi}, y = a \frac{\sin \xi \sin \varphi}{\cosh \eta - \cos \xi}, z = a \frac{\sinh \eta}{\cosh \eta - \cos \xi}, \quad (9)$$

the bulk elastic energy [13] of a tactoid equals to

$$\frac{a\gamma}{2} \int_0^{2\pi} d\varphi \int_{-\infty}^{\infty} d\eta_{kazn} \int_{-\pi}^{\pi} \frac{4K_1 \sinh^2 \eta_{kazn} \sin \xi + K_3 \sin^3 \xi}{(\cosh \eta_{kazn} - \cos \xi)^3} Dd\xi. \quad (10)$$

The corresponding vector potential is

$$V(\vec{\phi}) = \frac{1}{2}\lambda + (\vec{\phi}^+ \cdot \vec{\phi} - \frac{1}{2}\zeta^2)^2 + \frac{1}{2}k|\vec{\phi} \cdot \vec{\phi}|^2. \quad (11)$$

At the parameter $k > 0$, the vacuum is characterize by $\vec{\phi} \cdot \vec{\phi}, \vec{\phi}^\dagger \cdot \vec{\phi} = \frac{1}{2}\zeta^2$.

$$\Phi_0 = \frac{\zeta}{2} \begin{pmatrix} -\frac{\sin \xi \sinh \eta_{kazn} \cos \varphi_{kazn}}{\cosh \eta_{kazn} - \cos \xi} \\ -\frac{\sin \xi \sinh \eta_{kazn} \sin \varphi_{kazn}}{\cosh \eta_{kazn} - \cos \xi} \\ \frac{1 - \cosh \eta_{kazn} \cos \xi}{\cosh \eta_{kazn} - \cos \xi} \end{pmatrix}. \quad (12)$$

The generators of $SU(2)$ are denoted as $T_{i=1,2,3}$. T_0 is the generator of $U(1)$. $-iT_1(\vec{\phi}_j) = -\epsilon_{ijk}\vec{\phi}_j$, $-iT_0(\vec{\phi})_j = -\vec{\phi}_j$. $Q = T_2 + T_0$ is the annihilation condition. The string generator ($T_S = T_3$) does not commute with the charge generator: $[T_S, Q] = [T_3, T_2] = -iT_1$ [22]. Here $\vec{\phi}(\alpha) = e^{-i\alpha T_3}\vec{\phi}$ are also meaning the generators. Between the vacuums, the angular dependence is established $Q(\theta) = e^{-i\theta T_3}Qe^{i\theta T_3}$ [29]. Tactoid vortices revolve $SU(2)$. R, α are introduced to describe the tactoid geometry (Fig.1).

Further, we need to solve the next equations of motion:

$$\frac{1}{\sqrt{g}}\partial_\mu\sqrt{g}\mathbf{F}^{\mu\nu} = j^\alpha = je[\vec{\phi}^\dagger \times D^\nu\vec{\phi} - \vec{\phi} \cdot (D^\nu\vec{\phi})^\dagger], \quad (13)$$

$$\frac{1}{\sqrt{g}}\partial_\mu\sqrt{g}\mathbf{G}^{\mu\nu} = \mathbf{J}^\alpha = g[\vec{\phi}^\dagger \times D^\nu\vec{\phi} + \vec{\phi} \times (D^\nu\vec{\phi})^\dagger], \quad (14)$$

$$\frac{1}{\sqrt{g}}D_\mu\sqrt{g}\vec{\phi} = \frac{\delta V}{\delta\vec{\phi}^\dagger}. \quad (15)$$

To confirm the existence of vortons, labeled by vacuum, and estimate the energy T_2 , the first-order Bogomol'ny's equations are usually applied. the first-order Bogomolny's equations are usually applied. For example, in the sigma-model limit of the Lagrangian of the type (5), the non-Abelian vortons with the (1, 1)-, (1, 2)- and other pairs of winding numbers in $SU(2)$ were numerically revealed by Radu and Volkov [21] just lately; to be solvable, their model has included four free parameters in the potential (Fig. 2).

There was numerically proven with help of Gauss-Tschebuechev algorithm, that in the $U(1)$ gauge, the stable vortons may appear [27], whereas in $SO(3)$ it is not so [26]. The stability criterion includes the radius R of the vortex loop, which may be compared with the Kaznacheev-van der Schoot theoretical analysis [2], [7], and with the lattice Monte Carlo simulations, performed by Bates [11].

IV. DYNAMICS AND ESTIMATIONS FOR TIME-AGING OF THE TACTOID SOLS

Along with these assertions on the configurations supplied with Non-Abelian gauge fields, the approximate methods of analysis exist for quite attainable numerical simulations of vorton states. One of there is so called Abelian projection [33]. So, following the Maximal Abelian (MaA) projection approach, we fix $SU(2)$ gauge and leave the winding group $U(1)$ unfixed. In applied numerical tasks, Abelian approximations of (11) are yet acceptable.

For example, whether is an analogous Abelian projection of the $V_2O_5 - H_2O$ tactoid configuration realized in the 2D ferromagnetic systems and thin films [34], if there

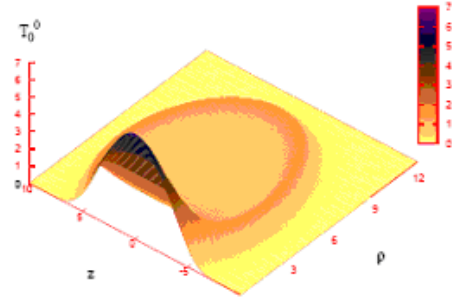


FIG. 2: The energy density of $n=m=1$ vortons [21], plotted by Radu and Volkov numerically at four free parameters, where z and ρ are the polar coordinates.

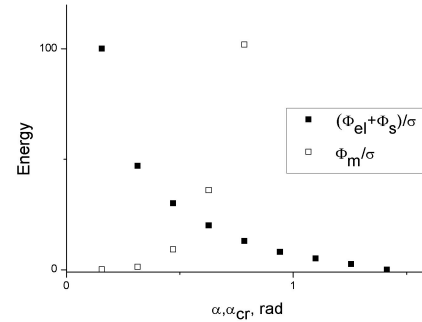


FIG. 3: Competition between the magnetic Φ_m and the elastic and the surface energies $\Phi_{el} + \Phi_s$ of a tactoid, divided by $\sigma = 10^{-3}$ erg/cm, errors are not indicated; as this is a qualitative view of (3) and experiments [2], [12]; the data at $\gamma \rightarrow 1$, from which $\alpha \approx 32^\circ$.

are defined the same topological invariants? This simplification is useful to estimate the case of annihilating particles with whole unit opposite charges [3]. One may express the vorton dynamics by the Landau-Lifshitz equation (LLE), including dissipation (labeled by the constant α [34]), and write:

$$\frac{\partial \mathbf{m}}{\partial t} = \mathbf{m} \times \mathbf{f} - \alpha_d \mathbf{m} \times (\mathbf{m} \times \mathbf{f}), \mathbf{f} \equiv \Delta \mathbf{m} - Q m_z \hat{\mathbf{e}}_z, \quad (16)$$

where \mathbf{m} is the magnetization vector, Q is the free parameter, $\hat{\mathbf{e}}_z$ is the unit vector in the z magnetization direction. According to the definition [3], the topological invariant N connected with the topological density n is $N = \frac{1}{4\pi} \int_V n \epsilon_{\mu\nu} \mathbf{r}^3$, $\epsilon_{\mu\nu}$ is the asymmetric tensor with $(\mu, \nu) = (1, 2)$, V is a tactoid volume, and the vector \mathbf{r} denotes its space.

The magnetic field stretches large tactoids (a increases), whereas to annihilate, the tactoid shape should become more oblate [12], Fig. 3. Therefore, the equilibrium angle α , corresponding to the large tactoid shape,

exists also for coalescence in the magnetic field. To define a is not difficult from the next simple algebra with (1) and (2), by using the definitions [3, 10-11]. From (1) - (3) and (12), dynamics characteristics of a solitary tactoid may be expressed as:

$$\frac{1}{(\chi H)^4} \left(\frac{\partial \mathbf{m}}{\partial t} \right)^2 \sim \alpha_d^2 \left(\frac{1 - \cosh \eta_{kazn} \cos \xi}{\cosh \eta_{kazn} - \cos \xi} \right)^2. \quad (17)$$

E. g. stretch of a tactoid in z -axis direction increases its magnetic energy, and the magnetic field is precipitating for annihilation of droplets, as a free volume decreases.

On the experiment [14], the next parameters are measured: $C_i = \frac{K_i}{\sigma}$, $i = 1, 3$, K_i are moduli of (2), and σ is the surface tension. C_3 -s order is hundreds micrometers. For C_1 -s, these are about unit. Both of them are drop-down with time, but according to (1)-(2), have not affect on the magnetic term.

V. CONCLUSIONS

We composed the topological classification of sols $V_2O_5 - H_2O$, owing to which, the qualitative practical

predictions for thermodynamic states of these sols may be performed. The cosmological theory of superconductive strings supposes that the nematic tactoids in $V_2O_5 - H_2O$ annihilate in accordance with non-Abelian statistics.

This process, carried out in magnetic field, increases a time-aging of the sols, but does not yield to direct exact estimations, since its nature is principally Non-Abelian. One may connect an actual electromagnetic interaction in the $V_2O_5 - H_2O$ solution via pH value and discuss questions on the tactoid junction in frames of chemistry, which we have wittingly ignored in favor of the important topological role. The process of tactoid junction in magnetic field leads to rise of the additional electromagnetic field changing pH of water around tactoids and, for one's part, time-aging [2, 13]. These observations may be important for ecology, as long as vanadium pentoxide is contained in impurities of coal soles, which are the components of wastes of thermoelectric power stations and are included in the impurity parameters at the background control for radiation.

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